Starter Questions

Find the equation of a line with gradient 3 that goes through the point . Give your answer in the form
where , and are integers.

Find the equation of a line perpendicular to
that goes through the point . Give your answer in the form where , and are integers.

G Differentiation

G1

Understand and use the derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x and for $\sin x$ and $\cos x$

Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

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4.3 Rates of Change

$\frac{dy}{dx}$ can be referred to as the 'rate of change of with respect to '

The gradient of a distance-time graph is a measure of the rate of change of distance (displacement, r) with respect to time, this is called speed (velocity, v).

Hence, .

The rate of change of velocity is called acceleration.

Hence .

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4.3 Rates of Change

Example 1

A particle is moving on the -axis such that its distance (displacement), cm, from the origin is given by , where is the time measured in seconds.

a) Use the fact that the velocity to find an expression for the particle’s velocity.

$$v = \frac{dr}{dt} = 3t^2 + 4t + 1$$

4.3 Rates of Change

Example 1

A particle is moving on the -axis such that its distance (displacement), cm, from the origin is given by , where is the time measured in seconds.

b) Use the fact that the acceleration to find an expression for the particle's acceleration.

$$a=\frac{dv}{dt}=6t+4$$

4.3 Rates of Change

Example 2

The volume, V m^3, of water in a tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2 \text{ for } t \geq 0$$

(a) Find $\frac{dV}{dt}$

(b) Find the rate of change of the volume of water in the tank, in m^3 s^{-1}, when $t = 2$

$$\frac{dV}{dt} = 2t^5 - 8t^3 + 6t$$

$$\frac{dV}{dt} = 2(2)^5 - 8(2)^3 + 6(2) = 12\, m^3 s^{-1}$$

4.3 Rates of Change

Example 3a

A model helicopter takes off from a point O at time $t = 0$ and moves vertically so its height, y cm, above O after time t seconds is given by:

$$y = \frac{1}{4}t^4 - 26t^2 + 96t\ ,\ 0 \le t \le 4$$

(a) Find $\frac{dy}{dt}$

$$\frac{dy}{dt} = t^3 - 52t + 96$$

4.3 Rates of Change

Example 3c

A model helicopter takes off from a point O at time $t = 0$ and moves vertically so its height, y cm, above O after time t seconds is given by:

$$y = \frac{1}{4}t^4 - 26t^2 + 96t\ ,\ 0 \le t \le 4$$

(c) Find the rate of change of y with respect to t when $t = 1$

$$\frac{dy}{dt} = t^3 - 52t + 96$$

cm/s

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4.3 Rates of Change

Example 3d

A model helicopter takes off from a point O at time $t = 0$ and moves vertically so its height, y cm, above O after time t seconds is given by:

$$y = \frac{1}{4}t^4 - 26t^2 + 96t\,,\ 0 \le t \le 4$$

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$

$$\frac{dy}{dt} = t^3 - 52t + 96$$

the height is decreasing when

Complete the rates of change questions

G	Differentiation
G3	Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points, points of inflection. Identify where functions are increasing or decreasing.

https://sites.google.com/view/tlmaths/home/a-level-maths/as-only/g-differentiation/g3-gradients#h.prqo2x4z9pzf

G3-02, G3-03, G3-05

G Differentiation

Students should:

- understand and be able to use the fact that at a stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
- describe a stationary point as a (local) maximum or minimum
- know that:

At a maximum $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} < 0$

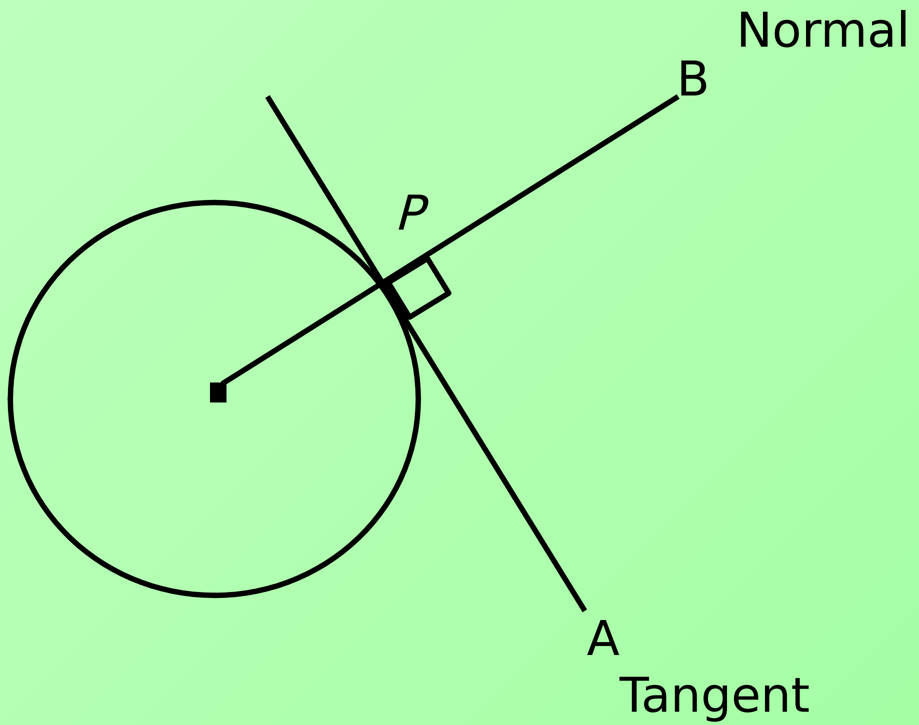
At a minimum $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} > 0$

Note:

the case $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$ will not be tested at AS

- use $m_1 \times m_2 = -1$ for gradients of tangent and normal
- be able to answer questions set in the form of a practical problem where a function of a single variable has to be optimised
- be able to show that a function is increasing or decreasing, by showing $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ or $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$ respectively.

4.4 Tangents and Normals



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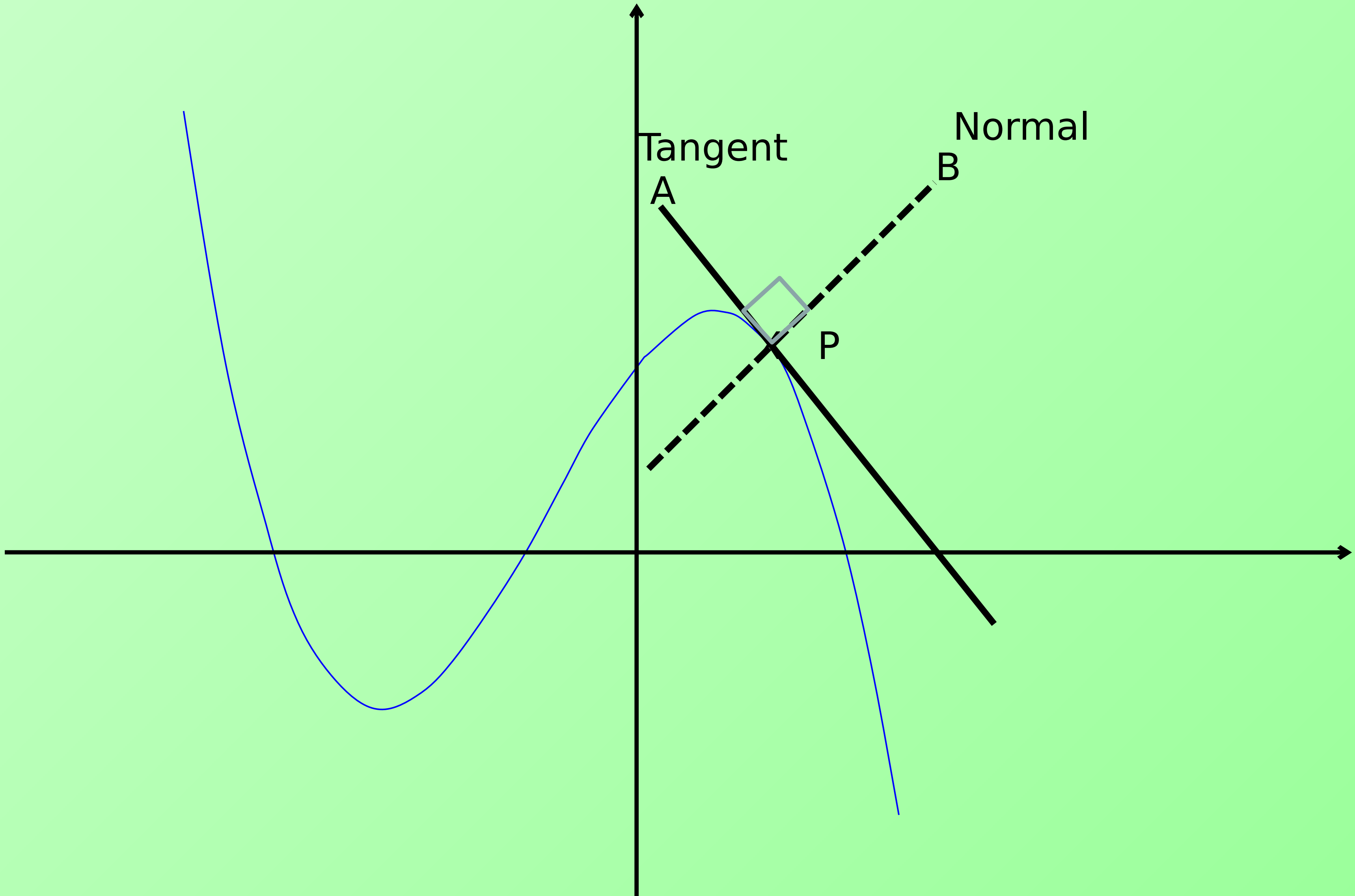
4.4 Tangents and Normals



4.4 Tangents and Normals

Example 1:

A curve has equation .
Giving your answers in the form
(where , and are integers) find the equation of:

i. the **tangent** at the point on the curve where

At

$$y=2^3-6(2)+1=-3$$

4.4 Tangents and Normals

Example 1:

A curve has equation .
Giving your answers in the form
(where , and are integers) find the equation of:

ii. the **normal** at the point on the curve where

Normal and tangent are perpendicular

4.4 Tangents and Normals

You try: Example 2:

Find the equation of the TANGENT to the curve at the point

At

4.4 Tangents and Normals

You try: Example 3:

Find the equation of the NORMAL to the curve when .

At

$$y = 6^2 - 5(6) = 6$$

$$m_T = 7, \therefore m_N = -\frac{1}{7}$$

4.4 Tangents and Normals

Example 4:

The line is a tangent to the curve

,

$$\square_{\square} = \mathbf{3}$$

a) Work out the point where the tangent meets the curve, thus find the value of the constant b

$$\therefore 2 + \frac{2}{\sqrt{x}} = 3$$

Meets the curve at (4, 16)

4.4 Tangents and Normals

Example 4:

The line is a tangent to the curve

,

a) Work out the point where the tangent meets the curve, thus find the value of the constant b

Meets the curve at (4, 16):

4.4 Tangents and Normals

Example 4:

The line is a tangent to the curve

,

b) Work out the equation of the normal to the curve at this point

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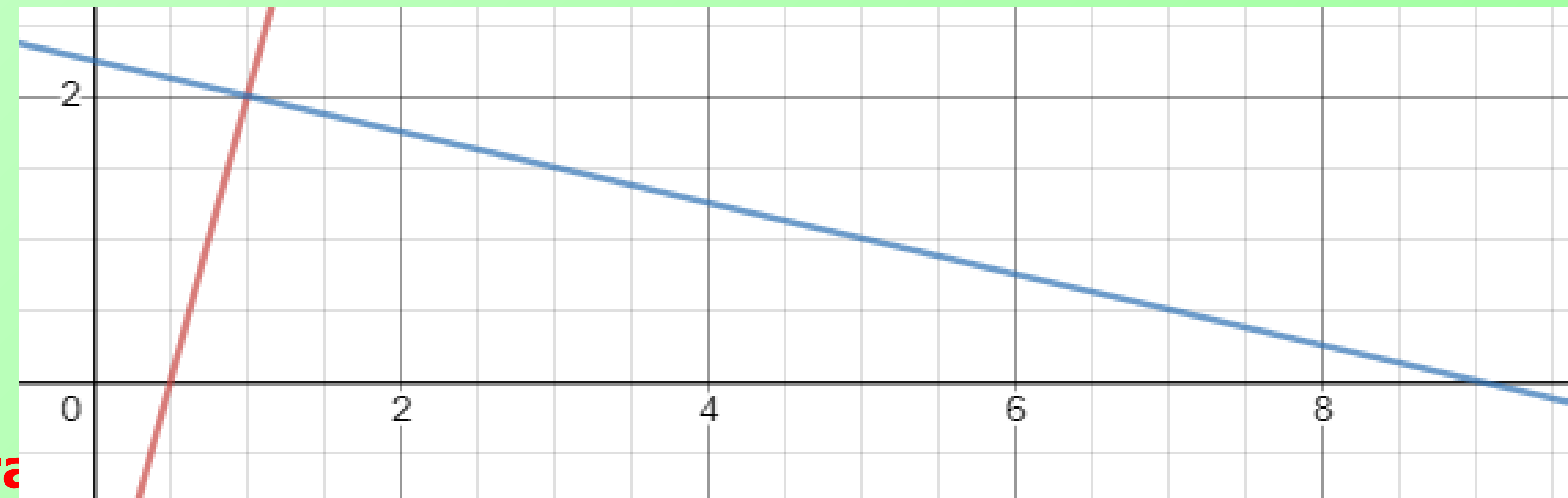
4.4 Tangents and Normals

Example 5:

The point T (1, 2) lies on the curve $y = x^3 + x$

Work out the triangular area trapped between the tangent and the normal to the curve at the point T and the x-axis



4.4 Tangents and Normals

Example 5:

The point T (1, 2) lies on the curve $y = x^3 + x$

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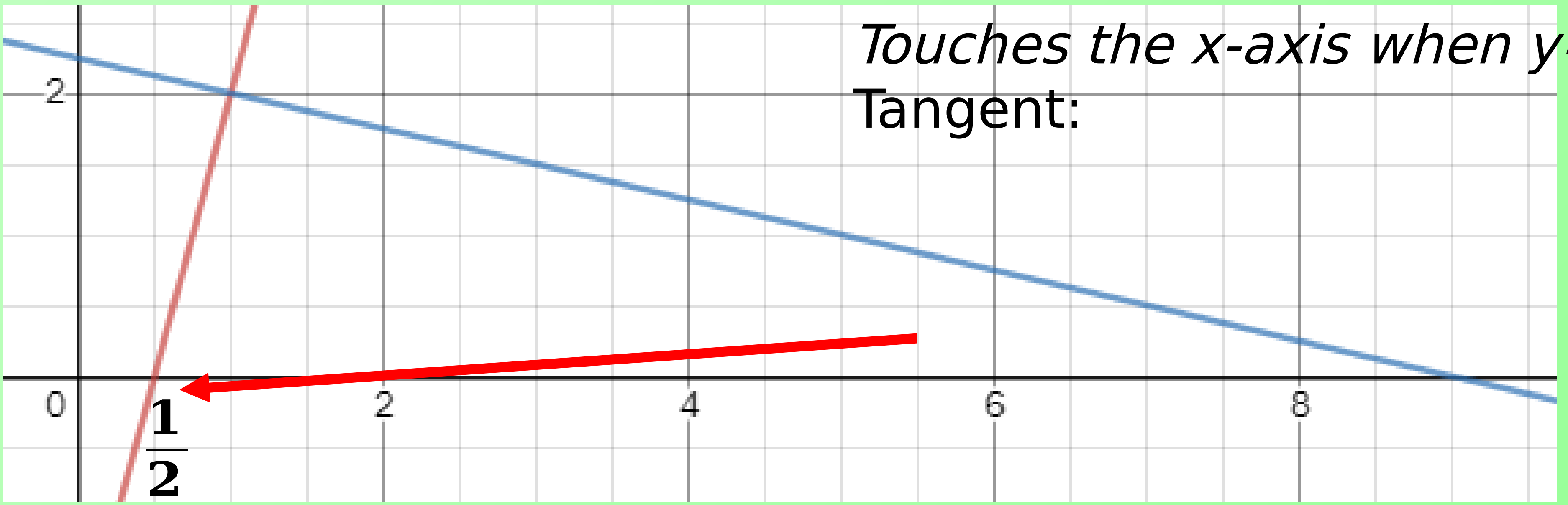

4.4 Tangents and Normals

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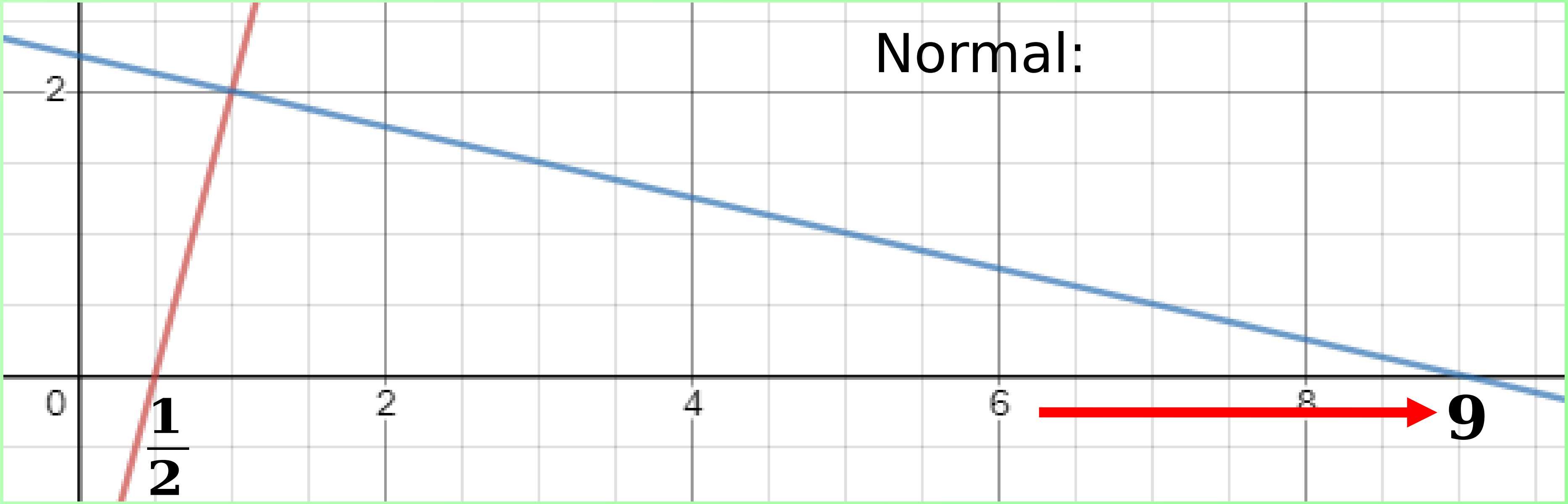

4.4 Tangents and Normals

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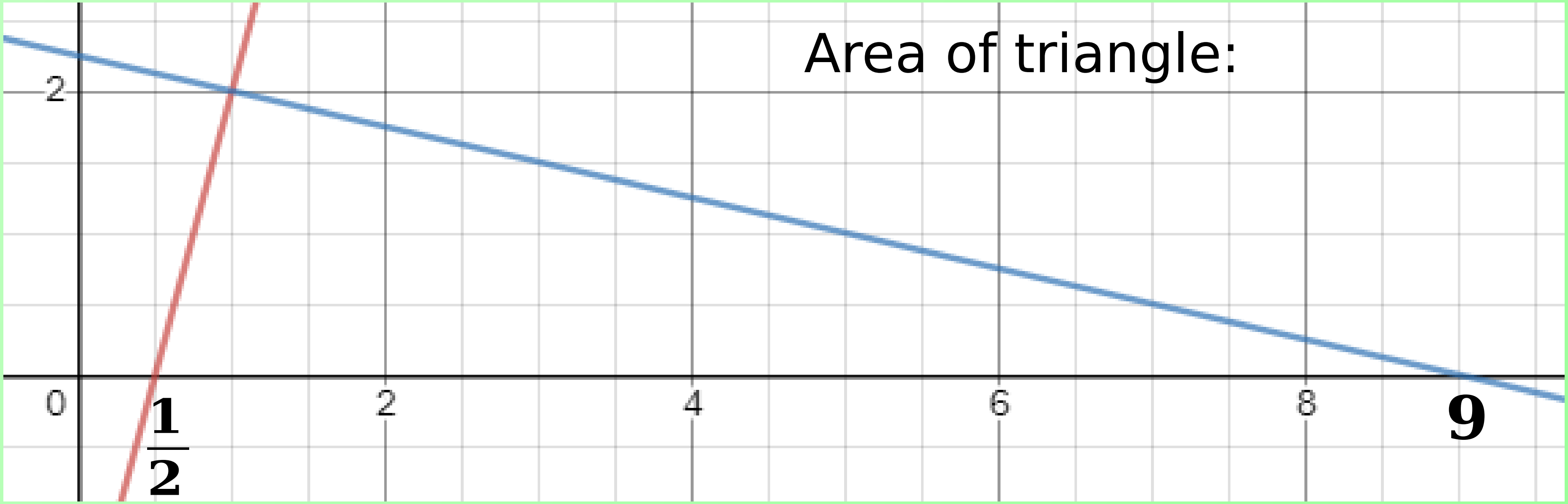

4.4 Tangents and Normals

To find the equation of a tangent/normal to a curve:

1. Differentiate the function to find
2. Find the gradient of the tangent/normal when

3. Substitute into to find the corresponding y-coordinate
4. Write the equation using:

Tangent:

Normal:

5. Rearrange to the required form if the question specifies, e.g.

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4.4 Tangents and Normals

Worksheet Qs

Bronze: Start at Q1

Silver: Start at Q2

Gold: Start at Q3

Answers on teams

Extension & exam question on back – answers on next slides

12 A curve has equation $y = 6x\sqrt{x} + \dfrac{32}{x}$ for $x > 0$

12 (a) Find $\dfrac{dy}{dx}$

[4 marks]

12 (b) The point *A* lies on the curve and has x-coordinate 4

Find the coordinates of the point where the tangent to the curve at *A* crosses the x-axis.

[5 marks]

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Rewrites given expression with a fractional power and negative power – at least one index form must be correct	AO1.1a	M1	$y = 6x^{\frac{3}{2}} + 32x^{-1}$
	Both terms correct	AO1.1b	A1	
	Differentiates 'their' rewritten expression – at least one term correct	AO1.1a	M1	$\frac{dy}{dx} = 6 \times \frac{3}{2} \times x^{\frac{1}{2}} - 32x^{-2}$ $= 9\sqrt{x} - \frac{32}{x^2}$
	Both terms correct for 'their' expression	AO1.1b	A1F	

(b)	Finds the equation of the tangent, a clear attempt must be seen	AO3.1a	M1	When $x = 4$,
	Evaluates 'their' $\frac{dy}{dx}$ (from part (a)) correctly (when $x = 4$)	AO1.1b	A1F	$\frac{dy}{dx} = 9 \times 2 - \frac{32}{16} = 16$ and
	Obtains correct y value (when $x = 4$)	AO1.1b	A1	$y = 6 \times 4 \times 2 + \frac{32}{4} = 56$
	Obtains correct form of the equation of a straight line using 'their' values for y and $\frac{dy}{dx}$	AO1.1b	A1F	Tangent: $y - 56 = 16(x - 4)$
	Deduces value required at x-axis is when y equals 0 (follow through from 'their' equation) Both coordinates needed, any form	AO2.2a	A1F	When $y = 0$, $x = 4 - \frac{56}{16} = 0.5$ (0.5, 0)
	Total		9	

Extension

A curve has the equation $y = x^3 - px + q$. The tangent to this curve at the point (2, -8) is parallel to the x-axis.

a) Find the values of p and q.

b) Find also the coordinates of the other point where the tangent is parallel to the x-axis.

xtension Answer

A curve has the equation $y = x^3 - px + q$. The tangent to this curve at the point (2, -8) is parallel to the x-axis.

a) Find the values of p and q.

b) Find also the coordinates of the other point where the tangent is parallel to the x-axis.

a) $p=12$; $q=8$

b) (-2, 24)